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I. Introduction

The concept of Forecast Scheduling (FS) has been introduced in [1], where it is shown that the knowledge of the present and future rates along the users' trajectories can be exploited by the scheduler in order to significantly improve the average user throughput. The throughput gain is achieved by exploiting long term time and spatial diversity along the users' trajectory. The FS is posed as a convex optimization problem that can be solved using fast convex optimziation solvers. The randomness of the traffic, i.e. arrival and departure of communications on the one hand and randomenss in the user trajectories can be incorporated into the FS solution [2].

This repport investigates the FS for the case of $n = 2$ users and derives closed form expressions for the FS allocation rules.

II. Forecast scheduling model and analytical results

In this Section we first briefly recall the basic formulation of the FS as presented in [1]. Then, using Karush-Kuhn-Tucker (KKT) conditions, we provide a closed form solution for $n = 2$ users with the main steps for deriving it.

A. Basic formulation

Consider a macro-cell (Base Station (BS)) surrounded by interfering BSs. We suppose that the Signal to Interference plus Noise Ratio (SINR) for the users in mobility at any location is provided to the BS by a Radio Environment Map (REM). Consider n full buffer users moving at a constant speed during a time interval T - the scheduling period, over which n is considered constant. Suppose that time is in a discrete space: $t \in \{1, 2, .., T\} = [|1, T|]$ and let i denote the user number, $i \in \{1, 2, .., n\} = [|1, n|]$. We suppose that during the scheduling duration there are no arrivals or departures of users (this assumption can be relaxed [2]).

A scheduling period T (typically of the order of seconds) is divided into scheduling time slots denoted here for simplicity as time units (e.g. of 1 ms), during which the bandwidth is shared among the scheduled users. Let $a_i(t)$ denote the bandwidth proportion allocated to a user i at time t, $a_i(t) \in [0, 1]$, according to the scheduling strategy, where $\forall t$, $\sum_{i=1}^{n} a_i(t) = 1$, and W - the total bandwidth. Using the Shannon equation, we write the throughput as a function ϕ of the SINR of user i as follows

$$a_i(t)\phi(SINR_i(t)) = a_i(t)Wlog_2(1 + SINR_i(t)). \quad (1)$$

Denote by S_i^t the predicted SINR (i.e. the one provided by the REM). The FS allocation policy is defined by the following optimization problem, with $\alpha \neq 1$:

$$\begin{aligned} &\text{maximize} : f(a) = \sum_{i=1}^{n} \frac{(\sum_{t=1}^{T} a_i(t)\phi(S_i^t))^{1-\alpha}}{1-\alpha} \quad (2)\\ &\text{subject to} : \forall i, \forall t, a_i(t) \geq 0\\ &\forall t, \sum_{i=1}^{n} a_i(t) = 1 \end{aligned}$$

and for $\alpha \to 1$, the optimization problem with the same constraints reads:

$$\text{maximize} : f(a) = \sum_{i=1}^{n} log(\sum_{t=1}^{T} a_i(t)\phi(S_i^t)). \quad (3)$$

Both equations (2) and (3) have concave functions f for $\alpha \geq 0$, and can be solved using convex optimization. The size of the optimization problem is defined by the number of unknown variables, namely $n \times T$.

The interpretation of (2) and (3) is the following: resources are shared fairly among the users according to the data-rate variation in their future trajectories. For example, if a user has a large enough coverage hole in his future trajectory, the forecast scheduler will take this into account and may allocate this user as much data as possible before reaching the coverage hole so as to remain fair with respect to the other users.

B. Analytical model for $n = 2$ two users

We derive presently the KKT conditions for the problem (2). The optimization problem (2) has a nonlinear objective function f with regular equality and inequality conditions i.e. differentiable constraint functions. The objective and constraint functions (2) are continuously differentiable at any $a = (a_1(1), a_1(2), ...a_1(T), a_2(1), ..., a_n(T)) \in \mathbb{R}^{nT}$, then there exist multipliers $\lambda_{k,j}$ and ν_j, where $k \in [|1, n|]$ and $j \in [|1, T|]$, called KKT multipliers ([3], Chap.5) with the following Lagrangian function:

$$L(a, \nu, \lambda) = f(a) + \sum_{j=1}^{T} \nu_j (\sum_{k=1}^{n} a_k(j) - 1) + \sum_{k,j} \lambda_{k,j} a_k(j), \tag{4}$$

where $\lambda_{k,j} \geq 0$.

We define the Lagrange dual function as the maximum value of L over a. Let a^* maximizes the Lagrangian function (4) for the optimal multipliers $\lambda^*_{k,j}$ and ν^*_j, where $k \in [|1, n|]$ and $j \in [|1, T|]$, therefore its gradiant is null at this point:

$$\nabla L(a^*, \nu^*, \lambda^*) = 0. \tag{5}$$

From (5) one obtains for all $i \in [|1, n|]$ and $t \in [|1, T|]$ the KKT conditions:

$$\begin{aligned} \frac{\partial f(a)}{\partial a_i(t)} + \frac{\partial}{\partial a_i(t)} \sum_{j=1}^{T} \nu_j (\sum_{k=1}^{n} a_k(j) - 1) + & \\ \frac{\partial}{\partial a_i(t)} \sum_{k,j} \lambda_{k,j} a_k(j) &= 0, \\ a_i(t) &\geq 0, \\ \sum_{i=1}^{n} a_i(t) &= 1, \\ \lambda_{i,t} a_i(t) &= 0, \\ \lambda_{i,t} &\geq 0, \end{aligned}$$

which is equivalent to :

$$\phi(S_i^t)(\sum_{j=1}^{T} a_i^*(j)\phi(S_i^j))^{-\alpha} + \nu_t^* + \lambda_{i,t}^* = 0, \tag{6}$$

$$a_i^*(t) \geq 0, \tag{7}$$

$$\sum_{k=1}^{n} a_k^*(t) = 1, \tag{8}$$

$$\lambda_{i,t}^* a_i^*(t) = 0 \tag{9}$$

$$\lambda_{i,t}^* \geq 0. \tag{10}$$

We note from (6) that at any time t, for all users i and w with $w \neq i$

$$\begin{aligned} &\phi(S_i^t)(\sum_{j=1}^{T} a_i^*(j)\phi(S_i^j))^{-\alpha} + \lambda_{i,t}^* \\ &\quad = \phi(S_w^t)(\sum_{j=1}^{T} a_w^*(j)\phi(S_w^j))^{-\alpha} + \lambda_{w,t}^* \end{aligned} \tag{11}$$

and also from (6), for all user i, we have for all times t and u:

$$\frac{\lambda_{i,t}^* + \nu_t^*}{\phi(S_i^t)} = \frac{\lambda_{i,u}^* + \nu_u^*}{\phi(S_i^u)} \tag{12}$$

as ν_t^* does not depend on the users, and $\sum_{j=1}^{T} a_i^*(j)\phi(S_i^j)$ does not depend on the time. The equality (11) explicits the resource balancing among users at each time relatively to α in the sense of equalizing the two expressions of each user.

C. KKT resolution for the case of two users

In all the calculations that follow, we rewrite $a = a^*$, $\lambda = \lambda^*$ and $\nu = \nu^*$.

Suppose there are two users in the cell, i.e. $n = 2$ in problem (2). There are still more unknown variables than equations. We introduce the following property to allow the resolution of the problem:

1) Problem assumptions: With some data rates condition, if the two (all, for the general case) users are scheduled simultaneously at some time $t = K$ then only one user is scheduled in all other times within $[1, T]$ and different from K

Theorem 1. ***If*** *the following conditions are verified:*
1/ There exists a time $t = K$ where $\forall i, a_i(K) > 0$ i.e. both users are scheduled simultaneously at time K,
2/ $\forall i, j$, $\frac{\phi(S_i^t)}{\phi(S_i^k)} \neq \frac{\phi(S_j^t)}{\phi(S_j^k)}$,
Then *$\forall t \neq K$ in $[1, T]$, there exists only one $j(t)$ such as $a_{j(t)}(t) > 0$.*

See the proof in the annexe.

From Theorem 1, one can deduce that only two cases exist:

- both users are scheduled simultaneously at time K and in other times only one of them is scheduled;
- only one user is scheduled at each time i.e. no such time K exists.

The assumption 2 of Theorem 1 comes from the fact that the data rates fractions are almost surely not equal between users in reality as they have almost never the same trajectories at the same time.

It is noted that numerical experiments have shown that the case described in Theorem 1 does not occur often. In the next two subsections we check the existence of the time K.

2) If time K exists then users are scheduled at the same time K: We suppose in the following that time K exists and that at a given time $t \neq K$ the user 1 is scheduled (and hence user 2 is not scheduled) then from equation (12):

$$\lambda_{1,t} = 0, \lambda_{1,K} = 0 \Leftrightarrow \frac{\nu_K}{\phi(S_1^K)} = \frac{\nu_t}{\phi(S_1^t)} \tag{13}$$

$$\lambda_{2,t} \geq 0, \lambda_{2,K} = 0 \Leftrightarrow \frac{\nu_K}{\phi(S_2^K)} \geq \frac{\nu_t}{\phi(S_2^t)} \tag{14}$$

Denote by $\psi_i^t = \frac{\phi(S_i^t)}{\phi(S_i^K)}$, then:

$$\psi_1^t > \psi_2^t \tag{15}$$

The inequality is strict as assumed in 1/ of the Theorem 1. User 1 is scheduled at t with the condition of existence of a time K if and only if the three conditions below are verified:

$$\begin{aligned} &Inequality(15), \\ &a_1(t) + a_2(t) = 1, \\ &a_1(t), a_2(t) \geq 0. \end{aligned}$$

Theorem 2. *If we suppose that a time K exists then for all time $t \neq K$ the users 1 and 2 are scheduled with the following scheduling strategy:*

$$\begin{aligned} a_1(t) &= 1_{\psi_1^t \geq \psi_2^t} \\ a_2(t) &= 1_{\psi_2^t \geq \psi_1^t} \end{aligned} \tag{16}$$

We can now check if a certain K exists by verifying if for both users $a_i(K) > 0$ using Theorem 2. If it is not the case then K does not exist. $a_1(K) > 0$ and $a_2(K) > 0$ can be written as following (see within proof of Theorem 3 in Annexe):

$$a_1(K) = \frac{1}{\phi(S_1^K)}\Big(\big(\frac{-\nu_K}{\phi(S_1^K)}\big)^{-1/\alpha} - \sum_{t\neq K} \phi(S_1^t) 1_{\psi_1^t \geq \psi_2^t}\Big) > 0 \tag{17}$$

$$a_2(K) = \frac{1}{\phi(S_2^K)}\Big(\big(\frac{-\nu_K}{\phi(S_2^K)}\big)^{-1/\alpha} - \sum_{t\neq K} \phi(S_2^t) 1_{\psi_2^t \geq \psi_1^t}\Big) > 0, \tag{18}$$

where $\nu_K = -\frac{(\phi(S_1^K)^{1/\alpha-1} + \phi(S_2^K)^{1/\alpha-1})^\alpha}{(1+\sum_{t\neq K} \psi_1^t 1_{\psi_1^t \geq \psi_2^t} + \psi_2^t 1_{\psi_2^t \geq \psi_1^t})^\alpha}$.

Note that $a_2(t) = 1 - a_1(t)$.

Using $\phi(S_i^K)\psi_i^t = \phi(S_i^t)$, denoting by $\phi_1 = \phi(S_1^K)$ and $\phi_2 = \phi(S_2^K)$, and using (17) and (18), we can summarize the result in *Theorem* 3 below.

Theorem 3. *K exists if and only if the following two inequalities are verified:*

$$\begin{aligned} &\phi_2^{\frac{1}{\alpha}} \sum_{t\neq K} \phi(S_1^t) 1_{\psi_1^t > \psi_2^t} - \phi_1^{\frac{1}{\alpha}} \sum_{t\neq K} \phi(S_2^t) 1_{\psi_2^t > \psi_1^t} < \phi_1^{\frac{1}{\alpha}} \phi_2, \\ &-\phi_2^{\frac{1}{\alpha}} \sum_{t\neq K} \phi(S_1^t) 1_{\psi_1^t > \psi_2^t} + \phi_1^{\frac{1}{\alpha}} \sum_{t\neq K} \phi(S_2^t) 1_{\psi_2^t > \psi_1^t} < \phi_2^{\frac{1}{\alpha}} \phi_1. \end{aligned} \tag{19}$$

If one of the conditions of Theorem 3 is not verified then time K does not exist and another scheduling strategy (rule) should be derived as described presently.

3) If time K does not exist: We suppose in this section that at time $u = 1$ user 1 is scheduled and therefore $\lambda_{1,1} = 0$. We need to verify this assumption, namely that user 1 is indeed selected at time $u = 1$. For all time t, the sign of $\lambda_{1,t} - \lambda_{2,t}$ determines the scheduling decision: if it is positive then the user 2 is scheduled at time t ($\lambda_{1,t} \geq \lambda_{2,t} \iff a_2(t) \geq a_1(t)$ by equation (10)). One of the lambdas must be null and the other one positive for the case of two users.

With equation (12), as $\lambda_{1,1} = 0$, we have:

$$\lambda_{1,t} - \lambda_{2,t} = \nu_1 \psi_1^{t/1} - \nu_1 \psi_2^{t/1} - \lambda_{2,1} \psi_2^{t/1} \tag{20}$$

where $\psi_i^{t/u} = \frac{\phi(S_i^t)}{\phi(S_i^u)}$. We have then:

$$\lambda_{1,t} - \lambda_{2,t} = \nu_1(\psi_1^{t/1} - \psi_2^{t/1}) - \lambda_{2,1}\psi_2^{t/1} \tag{21}$$

As $\lambda_{2,1}\psi_2^{t/1} > 0$ we divide the equation (21) by $\lambda_{2,1}\psi_2^{t/1}$ and hence the sign studied is the same as the sign of $\frac{\frac{\psi_2^{t/1}-\psi_1^{t/1}}{\psi_2^{t/1}}}{-\frac{\lambda_{2,1}}{\nu_1}} - 1$. Note that from (6), ν_1 is negative.

If $\psi_2^{t/1} < \psi_1^{t/1}$ then $\lambda_{1,t} - \lambda_{2,t} < 0$ hence user 1 will be scheduled at time t. We cannot say more using this method.

Theorem 4. *If user 1 is supposed scheduled at time $u = 1$, then:*
$\psi_2^{t/1} < \psi_1^{t/1} \implies$ *user 1 is scheduled at time t.*

Denote by A_1^u and A_2^u the following sets:

$$\begin{aligned} A_1^u &= \{t \neq u, \psi_1^{t/u} > \psi_2^{t/u}\} \cup \{u\} \\ A_2^u &= \{t \neq u, \psi_1^{t/u} < \psi_2^{t/u}\} \end{aligned}$$

For $u = 1$ and using Theorem 4 i.e. if $t \in A_1$ then we have that user 1 is surely scheduled at time $t > 1$ with the assumption that the user is scheduled at time 1. We have the following result (with the corresponding proof in the Annexe):

Theorem 5. *User 1 is scheduled at time u if and only if:*

$$(\sum_{A_2^u} \phi(S_2^j))^\alpha > \max_{k \in A_1^u} \frac{\phi(S_2^k)}{\phi(S_1^k)} (\sum_{A_1^u} \phi(S_1^j))^\alpha \tag{22}$$

Equivalent scheduling rule can be written for user 2.

III. Annexe

Proof of Theorem 1:

Suppose that the two users are scheduled together at two time instants K and h, then using the equivalence (9) and (12) thus:

$$\frac{\nu_K}{\phi(S_1^K)} = \frac{\nu_h}{\phi(S_1^h)}$$
$$\frac{\nu_K}{\phi(S_2^K)} = \frac{\nu_h}{\phi(S_2^h)},$$

therefore:

$$\frac{\nu_K}{\nu_h} = \frac{\phi(S_1^K)}{\phi(S_1^h)} = \frac{\phi(S_2^K)}{\phi(S_2^h)} \tag{23}$$

The data rates are almost surely not equal between users, as two users will never have two same positions in space at the same time, hence they cannot be scheduled simultanesouly in more than one time instance.

Proof of Theorem 3:

If we suppose that the two users (1 and 2) are scheduled at some time K i.e. $a_i(K) \neq 0$ with $i \in \{1,2\}$, and that $T > 1$, then according to Theorem 2 for all times $t \neq K$ there is at most one user scheduled at time t. Let i_1 and i_2 be in $\{1,2\}$, and for all $i \in \{1,2\}$:

$$\begin{cases} (a_{i_1}(K)\phi(S_{i_1}^K) + \sum_{t=1, t\neq K}^T \phi(S_{i_1}^t) 1_{\psi_{i_1}^t > \psi_{i_2}^t})^{-\alpha} = -\frac{\nu_K}{\phi(S_{i_1}^K)} \\ \sum_{i=1}^2 a_i(K) = 1, \\ \lambda_{i,t} \geq 0, \\ \lambda_{i,t} a_i(t) = 0 \\ a_i(t) \geq 0. \end{cases}$$

Therefore:

$$a_{i_1}(K) = \frac{(-\frac{\nu_K}{\phi(S_{i_1}^K)})^{-\frac{1}{\alpha}} - \sum_{t=1}^T \phi(S_{i_1}^t) 1_{\psi_{i_1}^t > \psi_{i_2}^t})}{\phi(S_{i_1}^K)} \tag{24}$$

We know that:

$$\sum_{i=1}^2 a_i(K) = 1, \tag{25}$$

then we have:

$$\nu_K = -\frac{(\phi(S_1^K)^{1/\alpha - 1} + \phi(S_2^K)^{1/\alpha-1})^\alpha}{(1 + \sum_{t \neq K} \psi_1^t 1_{\psi_1^t \geq \psi_2^t} + \psi_2^t 1_{\psi_2^t \geq \psi_1^t})^\alpha}. \tag{26}$$

We can now check if really a certain K exists or not by verifying whether $a_i(K) > 0$ for $i = 1, 2$ supposing the scheduling allocation for $t \neq K$ satisfies eq.16 in Theorem 2. If it is not the case then K does not exist. $a_1(K)$ and $a_2(K)$ are then expressed as follows:

$$a_1(K) = \frac{1}{\phi(S_1^K)}((\frac{-\nu_K}{\phi(S_1^K)})^{-1/\alpha} - \sum_{t \neq K} \phi(S_1^t) 1_{\psi_1^t \geq \psi_2^t}) > 0 \tag{27}$$

$$a_2(K) = \frac{1}{\phi(S_2^K)}((\frac{-\nu_K}{\phi(S_2^K)})^{-1/\alpha} - \sum_{t \neq K} \phi(S_2^t) 1_{\psi_2^t \geq \psi_1^t}) > 0, \tag{28}$$

with ν_K defined in equation (26). Therefeore $a_2(K) > 0$ is equivalent to:

$$\frac{1 + \sum_{t \neq K} \psi_1^t 1_{\psi_1^t \geq \psi_2^t} + \psi_2^t 1_{\psi_2^t \geq \psi_1^t}}{\phi(S_1^K)^{1/\alpha-1} + \phi(S_2^K)^{1/\alpha-1}} > \frac{\sum_{t \neq K} \phi(S_2^t) 1_{\psi_2^t \geq \psi_1^t}}{\phi(S_2^K)^{1/\alpha}} \tag{29}$$

$$\iff$$

$$\frac{1 + \sum_{t \neq K} \psi_1^t 1_{\psi_1^t \geq \psi_2^t} + \psi_2^t 1_{\psi_2^t \geq \psi_1^t}}{\phi(S_1^K)^{1/\alpha-1} + \phi(S_2^K)^{1/\alpha-1}} > \frac{\sum_{t \neq K} \psi_2^t 1_{\psi_2^t \geq \psi_1^t}}{\phi(S_2^K)^{1/\alpha-1}} \tag{30}$$

We multiply the both sides of the inequality with the both denominators we have then:

$$\phi(S_2^K)^{1/\alpha-1}+\phi(S_2^K)^{1/\alpha-1}\sum_{t\neq K}\psi_1^t 1_{\psi_1^t\geq\psi_2^t}$$
$$-\phi(S_1^K)^{1/\alpha-1}\sum_{t\neq K}\psi_2^t 1_{\psi_2^t\geq\psi_1^t}>0.$$

Using $\phi(S_i^K)\psi_1^t=\phi(S_i^t)$, and denote by ϕ_1 for $\phi(S_1^K)$ and ϕ_2 for $\phi(S_2^K)$ the last inequality is equivalent to:

$$-\phi_2^{\frac{1}{\alpha}}\sum_{t\neq K}\phi(S_1^t)1_{\psi_1^t>\psi_2^t}+\phi_1^{\frac{1}{\alpha}}\sum_{t\neq K}\phi(S_2^t)1_{\psi_2^t>\psi_1^t}<\phi_2^{\frac{1}{\alpha}}\phi_1$$

and $a_1(K)>0$ is equivalent to:

$$\phi_2^{\frac{1}{\alpha}}\sum_{t\neq K}\phi(S_1^t)1_{\psi_1^t>\psi_2^t}-\phi_1^{\frac{1}{\alpha}}\sum_{t\neq K}\phi(S_2^t)1_{\psi_2^t>\psi_1^t}<\phi_1^{\frac{1}{\alpha}}\phi_2$$

If the inequalities (31) and (III) are verified then we have an expression for $a_1(K)$ and $a_2(K)$, and for all $t\neq K$ the user $i(t)$ scheduled is:

$$i(t)=argmax_{i\in\{1,2\}}\frac{\phi(S_i^t)}{\phi(S_i^K)}.$$

<u>Proof of Theorem 5:</u>

1/ We first demonstrate *"if user 1 is scheduled at time $u=1$ then we have the inequatiy of Theorem 5"*:

Using KKT condition with $\lambda_{2,1}\geq 0$ then for user 2:

$$\phi(S_2^1)(\sum_{j=1}^{T}a_2(j)\phi(S_2^j))^{-\alpha}+\lambda_{2,1}=-\nu_1, \qquad (31)$$

and for user 1 $\lambda_{1,1}=0$:

$$\phi(S_1^1)(\sum_{j=1}^{T}a_1(j)\phi(S_1^j))^{-\alpha}=-\nu_1, \qquad (32)$$

if $\lambda_{2,1}>0$, then

$$\frac{\phi(S_1^1)}{(\sum_{j=1}^T a_1(j)\phi(S_1^j))^\alpha}>\frac{\phi(S_2^1)}{(\sum_{j=1}^T a_2(j)\phi(S_2^j))^\alpha}, \qquad (33)$$

or written differently:

$$\phi(S_1^1)(\sum_{j=1}^{T}a_2(j)\phi(S_2^j))^{\alpha}>\phi(S_2^1)(\sum_{j=1}^{T}a_1(j)\phi(S_1^j))^{\alpha} \qquad (34)$$

Using the fact that if $\psi_1^t>\psi_2^t$ the user 1 is scheduled in time t. For sake of simplicity, we denote by $A_1=A_1^1$ and $A_2=A_2^1$, where $A_1^1=\{t>1,\psi_1^t>\psi_2^t\}\cup\{1\}$ and $A_2^1=\{t>1,\psi_1^t<\psi_2^t\}$. If $t\in A_1$ then user 1 is scheduled at time t, hence from inequality (34) we deduce:

$$\phi(S_1^1)(\sum_{A_2}a_2(j)\phi(S_2^j))^\alpha>$$
$$\phi(S_2^1)(\sum_{A_2}a_1(j)\phi(S_1^j)+\sum_{A_1}\phi(S_1^j))^\alpha. \qquad (35)$$

Since $0\leq a_2(j)\leq 1$:

$$\phi(S_1^1)(\sum_{A_2}\phi(S_2^j))^\alpha>\phi(S_2^1)(\sum_{A_1}\phi(S_1^j))^\alpha \qquad (36)$$

This inequality must also be valid for all $k\in A_1$ if user 1 is selected at time $t=1$:

$$\phi(S_1^k)(\sum_{A_2}\phi(S_2^j))^\alpha>\phi(S_2^k)(\sum_{A_1}\phi(S_1^j))^\alpha \qquad (37)$$

Consider next the case where $\lambda_{2,1}=0$ (rarely occurs when user 2 is not scheduled), therefore from equation (21):

$$\lambda_{1,t}-\lambda_{2,t}=\nu_1\psi_1^{t/1}-\nu_1\psi_2^{t/1}. \qquad (38)$$

From the sign of (38), user 1 is scheduled if and only if $t\in A_1$ and user 2 is scheduled at any $t\in A_2$. Using equations (31) and (32):

$$\frac{\phi(S_1^1)}{(\sum_{j\in A_1}\phi(S_1^j))^\alpha}=\frac{\phi(S_2^1)}{(\sum_{j\in A_2}\phi(S_2^j))^\alpha} \qquad (39)$$

We can write:

$$\frac{\phi(S_1^t)}{(\sum_{j\in A_1}\phi(S_1^j))^\alpha}=\frac{\phi(S_1^1)}{(\sum_{j\in A_1}\phi(S_1^j))^\alpha}\times\frac{\phi(S_1^t)}{\phi(S_1^1)} \qquad (40)$$

and a similar expression can be written for user 2. If $t\in A_1$ we have:

$$\frac{\phi(S_1^t)}{(\sum_{j\in A_1}\phi(S_1^j))^\alpha}>\frac{\phi(S_2^t)}{(\sum_{j\in A_2}\phi(S_2^j))^\alpha} \qquad (41)$$

2/ We demonstrate the reciprocal *"if we have the inequality of the theorem 5 then the user 1 is scheduled at time u=1"*:

$\Leftarrow$ we suppose that;

$$(\sum_{A_2}\phi(S_2^j))^\alpha>\frac{\phi(S_2^1)}{\phi(S_1^1)}(\sum_{A_1}\phi(S_1^j))^\alpha$$

and suppose that the user 2 is scheduled at time 1, therefore:

$$\phi(S_1^1)(\sum_{A_2\cup\{1\}}\phi(S_2^j))^\alpha<\phi(S_2^1)(\sum_{A_1-\{1\}}\phi(S_1^j))^\alpha$$

But we have:

$$(\sum_{A_2\cup\{1\}}\phi(S_2^j))^\alpha>(\sum_{A_2}\phi(S_2^j))^\alpha$$

and:

$$\phi(S_2^1)(\sum_{A_1-\{1\}} \phi(S_1^j))^\alpha < \phi(S_2^1)(\sum_{A_1} \phi(S_1^j))^\alpha$$

Thus,

$$(\sum_{A_2} \phi(S_2^j))^\alpha < \frac{\phi(S_2^1)}{\phi(S_1^1)}(\sum_{A_1} \phi(S_1^j))^\alpha$$

which is a contradiction, and therefore user 1 is scheduled if the inequality of theorem 5 is verified. The theorem is valid for all time u.

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